Wednesday, April 12

Final Exam Review

16 pages

↳ 1 cover page

↳ 6 sketch page

↳ 8 pages each

- $\boxed{\text{5 questions}}$
  - ↳ 2 ~ 3 parts

answer booklet

* Zc

- 250 marks

- questions booklet

sketch

→ name #

Id.

1 2

answer booklet

(do not sketch)

name.

data sheet

Q1. encoding

⑤

input					
	1	1	2	2	3

output						
	2	1	2	2	1	3

Math

$\forall i \cdot i \in \mathbb{N} \Rightarrow i \geq 0$

PlusCal

\A i \in Nat : i >= 0

① $len(input) = output[1] + output[3] + output[5]$

$\wedge$

$len(output) \Rightarrow$

$= 6$

② $len(output) = 6 \Rightarrow \forall i \cdot 1 \leq i \leq output[1] \Rightarrow input[i] = output[2]$

$\wedge$

$\wedge$

	1	2	3	4	5	6	7
input →	2	3	3	3	4	4	5

$i : 2$

$j : 4$

$$\forall k \cdot i \le k \le j-1 \Rightarrow input[k] = input[k+1]$$

$\Downarrow$

i, j denotes a valid block

$$\forall k \cdot i \le k \le j \Rightarrow input[k] = input[i]$$

$\forall m, n \cdot m, n$ denote a valid block $\wedge$ size of $m, n \le$ size of i, j

~~ARI~~ - $\phi_1 \Rightarrow \phi_2$

$p1 \Rightarrow p2$

$$\boxed{\begin{array}{c} \models \phi \\ G\,\phi \end{array}} \to \text{apply definitions}$$

$\phi \quad \phi \quad \neg\phi$

* (1) Which one is stronger?

$p \Rightarrow q$
$\equiv \neg p \vee q$

(2) Justify (formally).

$\cancel{p1}$ $\neg(p \vee q)$

$\equiv \{$ de morgan $\}$

$\hookrightarrow \neg(p \vee q) \equiv \neg p \wedge \neg q$

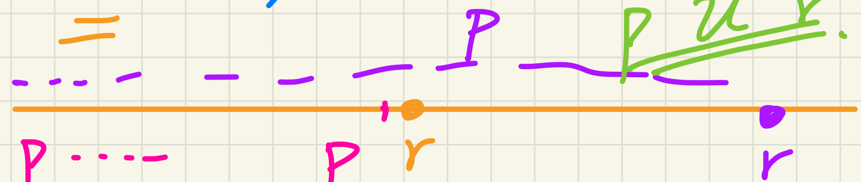
$\cancel{p2}$ $\neg p \wedge \neg q$.

$F(p\ \mathcal{U}\ r)$

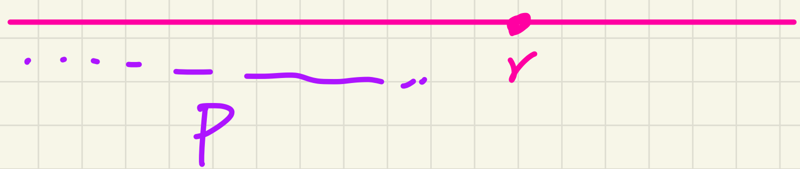
$\parallel$

$\cancel{p\ \mathcal{U}\ r}$

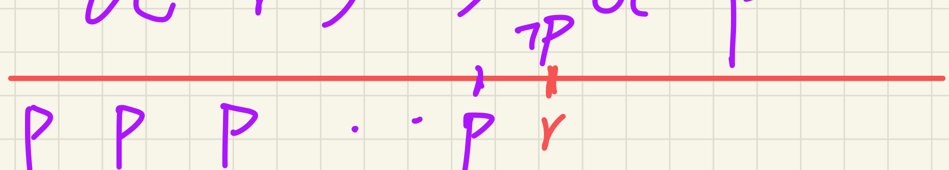
$G(p\ \mathcal{U}\ r)$



$p\ \mathcal{U}\ r$



$G(p\ \mathcal{U}\ r) \not\Rightarrow G\ p$



well-defined

dummy variable

$=$ {*wp* rule for assignment, twice}

$$input[i] > result \Rightarrow (\forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge \mathbf{input[i]} \ge input[j])$$
$$\wedge$$
$$input[i] \le result \Rightarrow (\forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge \mathbf{result} \ge input[j])$$

wp.

$Q \Rightarrow wp(S, R)$

$\{B \wedge Q\}\ S.\ \{R\}$

wp

We then prove that the precondition (i.e., Stay Condition $\wedge$ LI) is no weaker than the above calculated *wp*:

- To prove the left conjunct:

Q X

B

$$i \le Len(input) \wedge (\forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge result \ge input[j]) \Rightarrow$$
$$input[i] > result \Rightarrow \boxed{\forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge input[i] \ge input[j]}$$
$$\equiv \quad \{ \texttt{Shunting:} \quad p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r \quad \}$$
$$i \le Len(input) \wedge (\forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge result \ge input[j]) \wedge input[i] > result \Rightarrow$$
$$\boxed{\forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge input[i] \ge input[j]}$$

Prove $p \Rightarrow q$

Assum p, show q.

Proof via Assuming the Antecedent:

$$\boxed{\forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge input[i] \ge input[j]}$$
$$\equiv \quad \{\texttt{split range:}\ \forall j \bullet j \in 1\,..\,i \Rightarrow P(j) \equiv (\forall j \bullet j \in 1\,..\,i-1 \Rightarrow P(j)) \wedge P(i)\}$$
$$(\forall j \bullet j \in 1\,..\,\mathbf{i - 1} \Rightarrow 1 \le j \wedge j \le Len(input) \wedge input[i] \ge input[j]) \wedge (1 \le \mathbf{i} \wedge \mathbf{i} \le Len(input) \wedge input[i] \ge input[\mathbf{i}])$$
$$\equiv \quad \{\texttt{antecedent:}\ input[i] > \mathbf{result}\texttt{; and RHS of precond:}\ \forall j \bullet j \in 1\,..\,i \Rightarrow 1 \le j \wedge j \le Len(input) \wedge result \ge input[j]\}$$
$$true \wedge (1 \le i \wedge i \le Len(input) \wedge input[i] \ge input[i])$$
$$\equiv \quad \{\texttt{LHS of precond:}\ i \le Len(input)\ \texttt{and}\ input[i] \ge input[i] \equiv true\}$$
$$true$$

i valid index

LI: $\forall j \cdot \underline{j} \in 1\,..\,\underline{i} \Rightarrow input[i] \ge input[j]$

$p \Rightarrow (q \wedge r)$

$\equiv (p \Rightarrow q) \wedge \underline{(p \Rightarrow r)}$

-- algo.
assert - - ;

English → LTL

p : ___

q : ___

$p \, U \, q$

assert - - ;

LTL → English

$F G \phi$ -

$body

while (. . .) {

LI for outer loop

}

while (. .) {

LI for inner loop.

}

}